Welfare Comparison of Spectrum Property and Spectrum Commons Governance Regimes[*]

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I. Introduction

The continuing trend towards wireless communications in recent years has sparked a debate over the optimal spectrum governance regimes. On one side are the proponents of open commons, who claim that managing spectrum as a common pool resource would be the best solution to increase spectrum utilization and stimulate innovation. For example, technologists, such as Shepard (2002) and Reed (2002), and policy scholars, like Benkler (2003) and Werbach (2004), contend that the inclusive nature of the open commons regime will be a driving force towards more sophisticated devices and this trend will ultimately allow all people to access the spectrum as they please.

On the other hand, however, the property rights proponents predict tragedies under the open commons regime, arguing that exclusive rights to spectrum are required to ensure market efficiency and long run viability (Kwerel and Williams 2002, Hazlett 2001). Whereas Faulhaber and Farber (2003) and Hazlett (2001) acknowledge that a commons regime might work as long as spectrum is not scarce, they also point out that demand growth will eventually lead to scarcity. In their view, the solution to this resource allocation problem is institutional, rather than technical, measures.

The literature up to this point has pointed out the pros and cons of each regime, but the analyses tend to be highly conceptual, and we still do not have the analytical tools required for an overall evaluation and comparison of different regimes. As a first step towards an integrated comparative study, this paper aims at modeling the welfare characteristics of four possible governance regimes and using the model to evaluate static welfare performance of each. The regimes differ according to whether the number of competitors is determined by government or by open entry and whether the interference

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robustness of devices is set by regulators or determined by market forces alone. Central to the analysis is a formal description of demand for a wireless service, factoring in the effect of interference, which lies at the center of much of the controversy about spectrum management. With this framework, we can examine the welfare consequences of policymakers' choices regarding controls on entry and the engineering specifications of wireless devices.

The paper is organized as follows: Section II describes the properties of wireless communications and spectrum governance regimes addressed in the present study. This section is very important in that people from different disciplines think of wireless communications very differently. Economists tend to think about hypothetical, highly stylized settings, while engineers might start off with the goal of building a model reflecting the "real world", which tends to include more technical factors. Moreover, as a starting point, this paper aims at making an apples-to-apples regime comparison, abstracting from specific wireless services. It is thus important not to obscure this approach by drawing connections between wireless communication and certain types of technology (e.g., equating an open commons regime with WiFi). Section II therefore discusses in more detail the main questions and premises of this paper and identifies issues that it does not address. Section III introduces the basic economic model of demand for wireless communication services; Section IV incorporates engineering characteristics of wireless communications and further elaborates the model. With this model of demand for wireless services, in Section V we analyze the economic outcomes under different spectrum governing regimes. Section VI presents the results and Section VII concludes.

II. Essential Characteristics of Spectrum Appropriation and Governance

The spectrum debate involves two intertwining issues: the nature of demand for wireless services and institutional design that can affect players' behavior. In this section we identify factors that distinguish the demand for wireless services from the demands for other goods and services, as well as features that distinguish different regimes. These will be built into the baseline model.

Defining Feature of Demand for Wireless Services

The debate over spectrum management regimes centers around the issue of *interference*. Different regimes affect economic outcomes through different technical or institutional coordination (or, interference avoidance) mechanisms, which alter users' incentives and behavior when using wireless services. These coordination mechanisms and the consequent user behavior in turn determine the level of interference experienced by users and the economic benefits provided by wireless services. Therefore, a critical task in modeling market-level demand for wireless services is capturing the effect of interference on users' valuation of wireless services.

To make apples-to-apples comparisons, we assume a single service and technology associated with a given demand function (the derivation of the function will be presented in Sections III and IV) and focus on how different regimes affect performance. Because the effects of variation in technology on outcomes are not examined, we caution readers against using the results of this study to compare performance of real world services, such as Wi-Fi and cellular services, that differ in types of technologies employed as well as governance regime.

Also noteworthy is that at this point our model does not include network effects. Different wireless applications have different demand characteristics regarding the relationship between number of users and users' valuation of the services. For cellular telephony, the more users a network has the more valuable the service is to the users because they can reach more people. But not all wireless services display this positive externality. For example, wireless personal network (PAN) devices are used to replace wires between various devices. Network effects certainly can be an important feature of wireless communications and the inclusion of it is a prominent direction for future study. However, given the goal of identifying the defining feature(s), we leave out network effects in this baseline study.

Essential Features Distinguishing the Regimes

In general, what the commons proponents advocate is essentially an open access regime with minimum requirements (Anyone who buys a certified device can access the spectrum resource). Recognizing this distinction, we use the term "open commons" instead of simply "commons". Open commons proposals generally have the following characteristics: (1) Coordination (interference avoidance) is achieved through technical etiquettes built into devices (2) Subject to compliance with the technical etiquettes, there is no restriction on entry. As number of devices increases, users will experience greater interference since there are no other means (than the technical etiquettes) to guarantee that their messages will get through (3) A collective agency (probably consisting of equipment manufacturers and service providers) which determines the technical etiquettes, which can include a maximum power limit, communications protocols such as listen-before-talk, or a required level of interference robustness of the devices such as minimum signal-to-interference tolerance.

By contrast, a proposed property rights regime usually has the following characteristics: (1) The band for a service is sliced into blocks with exclusive usage rights. (2) The government determines the initial number of blocks and their assignments to users through titles of ownership or licenses. (3) Maximum acceptable interference levels are initially defined by the government and enforced by a combination of technical specifications and spectral separation between the signals of different service providers. For a band of fixed size, the maximum acceptable interference level determines an upper bound on the number of firms that can enter the market. Alternatively, the number of firms might be viewed as determining the minimum achievable interference. In the present study we assume the government sets the number of firms allowed to enter, since this interpretation is easier to model. Under some proposed property rights regimes, the spectrum owners can consolidate or further divide the spectrum blocks, and they may negotiate acceptable interference levels with their neighbors. These possibilities are not considered in this study.

Comparing these two sets of characteristics, we identify the following policy instruments as the major components of the regimes for the comparison:

(1) *Entry conditions*: while the open commons regime sets no limit on number of firms that can enter the market, the number of firms granted access to the spectrum under the property rights regime will be determined by the government. Importantly, these entry conditions reflect the extent to which users/firms will be protected from interference under different regimes.[1]

(2) *Interference robustness*: we will also look at a required level of interference robustness as a policy instrument. Many have argued that under an open commons regime firms will produce more sophisticated devices to cope with an increasingly noisy radio environment. By making interference robustness a policy instrument we can see whether mandated interference robustness level can improve welfare under the open commons regime. We can also examine the performance of a property rights regime under given mandated interference robustness levels.

III. III. Incorporating Interference in the Demand for a Wireless Service

Our goal in this and the following section is to present a model of demand for a wireless service that more fully incorporates the demand-influencing effects of interference than have the demand components of previous economic models of markets for wireless services. The model introduced shows that in addition to the obvious effect of reducing the utility realized by users, interference has other, less obvious, implications for the demand for a wireless service, particularly as it is perceived by service suppliers in competitive situations. This model of wireless demand is presented in this section and the one that follows and is incorporated in the models of competition in wireless markets under different governance regimes developed later in the paper. What will become clear is that the engineering properties of interference influence demand in a very complicated way. The full model of demand with interference is presented in Section IV. This section sets out critical assumptions underlying that model and illustrates the basic intuition motivating the model with a simplified representation of an inverse demand function that incorporates interference effects while suppressing most of the engineering details that add complexity to the full model presented in Section IV.

We model demand for a wireless service in a market served by n identical[2] firms, indexed by i and j ($1 \leq i, j \leq n$; $i \neq j$), with each firm operating at its own unique set of frequencies within the band allocated to the service. We assume each user acquires one wireless device and the cost of the device is included in the price charged by a service provider. We assume that in the absence of interference the service in question is homogeneous good and that the *sans* interference (SI) market demand for the service is linear. The SI inverse demand function is

$$P = a - bQ,$$

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[1] Given a specific demand function that properly incorporates interference effects on demand, determining the number of firms will also set the total number of devices sold (if firms are symmetric), which will in turn determine the interference level.

[2] Identical in the sense that these firms have identical production functions and cost structures so their decisions will be the same given the same condition.

where P is the market price, Q is the market quantity, and a and b are positive constants. The unit purchase assumption means each point on the inverse demand function represents the willingness-to-pay of a different potential buyer in the market.

Interference changes this picture in at least three ways. Most obvious is the suppression of demand for the service in general mentioned earlier. Less obvious is a peculiar form of differentiation among suppliers that arises because the interference that one supplier's devices causes for another supplier's customers varies inversely with the spectral distance between the frequencies at which they operate and directly with the number of customers served by the first supplier (given fixed power level). Thus an increase in the customer base for a spectrally distant competitor will produce a smaller reduction in demand for the service offered by representative firm i than would an equivalent increase in customers serviced by a spectrally closer competitor. Finally, the interference of a firm's devices with each other makes the demands experienced by individual suppliers more concave (or less convex) than they would be otherwise.

To incorporate interference in the demand function, we assume the reduction in utility due to a given amount of interference is the same for all users, regardless of their SI willingness-to-pay. A buyer chooses among suppliers by comparing the utility net of price delivered by each service and selecting the service for which this difference is largest. Because the effect of interference on utility is the same for all buyers, the rank ordering of suppliers will be the same for all buyers. Firms who tie for offering the best deal are assumed to divide the market evenly, which means that each firm in a market with n firms serves one n^{th} of the market. In the symmetric equilibria we will be examining, this means each firm will charge the same price. Symmetry in the equilibrium does not mean, however, that each firm sees itself as constrained to charge a price no higher than the market equilibrium price. Consider a representative buyer comparing the services of firms i and j. Let Γ_i be the utility loss due to interference associated with i's service and let Γ_j be the utility loss due to interference j's service. Similarly define P_i and P_j as the nominal prices charged by the two services and call $P_i+\Gamma_i$ and $P_j+\Gamma_j$ the interference-adjusted prices for the two services. For the consumer to be indifferent between the two services requires that $P_i+\Gamma_i = P_j+\Gamma_j$, not $P_i = P_j$. It is the interference-adjusted price charged by competitors, not their nominal price, that cannot be exceeded by any individual firm in a competitive wireless market. The utility loss due to interference experienced by a firm's users is influenced by the number of customers it serves (because they interfere with each other) and by the ability of its devices to distinguish intended messages from interference, a property we will refer to as robustness. Devices can be manufactured with varying degrees of robustness, but more robust devices are more costly. The fact that robustness is a choice variable means each firm is able to increase its nominal price without losing its customers if it is willing to incur the added expense of devices sufficiently robust to ensure that the price increase is matched by a corresponding reduction in interference utility loss experienced by its customers.

The models of competition under different governance regimes presented below assume wireless service suppliers are Cournot competitors, which means each supplier expects its competitors to respond to changes in its interference-adjusted price (IAP) with the adjustments in their IAPs to hold their customer counts at their original levels. Let Γ_i

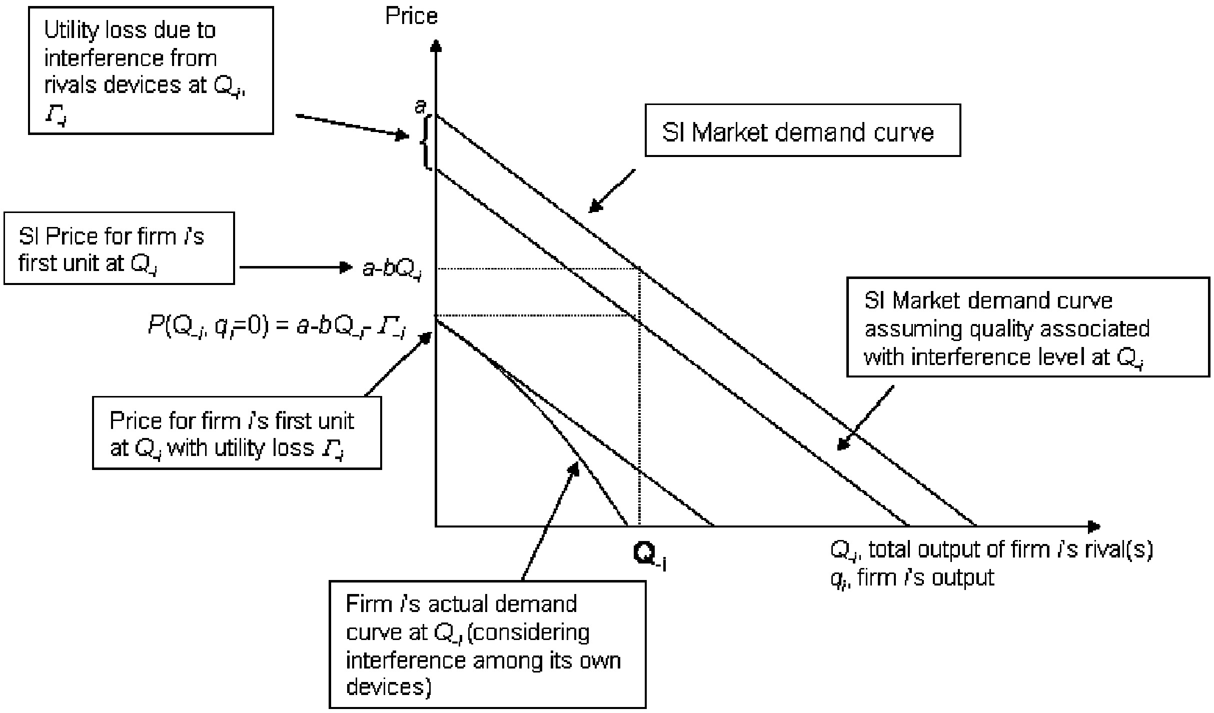
be the simple sum of separable utility-reducing effects of interference experienced by *i*'s users due to interference from other firms' devices and interference each *i* customer experiences from other *i* customers. For q_i the number of *i*'s customers, represent these two effects by Γ_{-i} and $\gamma_i(q_i)$, respectively, and let Q_{-i} be the sum of customers served by all firms but *i*. Clearly, $\gamma_i(q_i)$ is an increasing function of q_i. The inverse demand schedule perceived by *i* is described by equation (1).

$$P_i = a - b(q_i + Q_{-i}) - \gamma_i(q_i) - \Gamma_{-i} \qquad (1)$$

The construction and shape of this inverse demand function are described graphically in Figure 1.

In the next section we will specify the form of utility loss due to interference. Although the Γ and γ functions are useful for explaining the underlying intuition of the demand for wireless services, we will not use them in the next section because they will introduce unnecessary computation complexity.

Figure 1 An individual firm's demand curve



IV. The Fully-Specified Demand Function

This section fleshes out the model of demand introduced in Section III by more explicitly incorporating various engineering properties of signal propagation and interference suppressed in equation (1).

Section III pointed out that users' utility drops as they experience increasing level of interference given fixed transmission power level of the devices; this actually is derived from the fact that utility realized by a wireless service user increases with the intensity of the desired signal relative to aggregate interference at the receiver, everything else being the same. In addition to fixed transmission power, we also assume that utility falls inversely with a linear function of aggregate level of interference. These assumptions allow us to focus on aggregate interference and approximate the relationship between utility and interference by the following demand function

$$P_i = \frac{a}{1+[\beta_i(q_i)+\sum_{j,j\neq i}\beta_j(q_j)]} - b(q_i + Q_{-i}) \qquad (2)$$

where the $\beta_i(q_i)$ and $\beta_j(q_j)$ reflect the intensity of interfering signals (relative to the desired signal) from firm *i*'s own devices and any rival firm *j*'s devices, respectively, at a receiving device of firm *i*. We call these *interference intensity* functions.

Section III has already pointed out some of the properties the β functions should have: the interference intensity function is decreasing in the spectral distance between the interfered and interfering devices, as well as in the receiving device's interference robustness and cost. The aggregate interference intensity is the sum of interference intensity of individual devices and therefore is increasing in the number of interfering devices. To express these relationships in functional forms, we need to introduce some simplifying assumptions: each firm's devices are statistically uniformly distributed over the geographic service area (so that we do not need to worry about the issue of mobility), and that each firm chooses a unique sub-band for the operation of its devices within the spectrum allocated for this service, and that firms can costlessly relocate to other frequencies[3] (so the only concern they have when choosing operating frequency is to stay as far away from rivals in the frequency space as possible in order to minimize mutual interference).[4]

Let d_{ij} be the spectral distance between the operating frequency of firm *i* and that of firm *j*, *s* the geographically-averaged interference level contributed by each individual device, and $c(R)$ The cost of producing one device of robustness *R*. We assume $c'(R) > 0$ and employ the simplifying assumption that $c'(R) = 1$, so cost is also a direct measure of robustness.

For analytical convenience, we specify the interference intensity functions as inverse-proportionally decreasing in both the robustness of the device *c* and $(d_{ij}+1)$.[5] Given that interference intensity is contributed to by each individual device with average interference level *s*, we can then write the β functions as follows:

$$\beta_i = \frac{K_i \cdot q_i}{c}\,^{6} \text{ and } \beta_j = \frac{K_j \cdot q_j}{c \cdot d_{ij}},$$

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[3] When the firms are symmetric, this assumption will lead to even spacing between firms' operating frequencies, a property that significantly simplifies the analysis.

[4] We can illustrate the interference effect in the frequency domain by assuming that each of these wireless systems has only one channel. Although most modern systems have multiple channels, this simplification should not cause loss of insight. This is because we consider the frequency at which the interfering devices operate we will have to take the probabilistic average, which analytically will be the equivalent of the single channel system.

[5] Depending on the technology, these relationships can vary.

[6] To be accurate, β_i should be $\frac{K_i \cdot (q_i - 1)}{c}$, but this does not changes the results much except introducing technical complexity in implementing the calculations.

where K_i and K_j are constants, and K_i/s and K_j/s reflect the relative magnitudes of interference effects from firm *i*'s own devices and rivals' devices. These constants may be affected by firm-specific coordination techniques used to reduce interference among firms' own devices.

Apparent in these specifications is that as the number of devices increases, interference intensity goes up. On the other hand, when the level of interference robustness or distance in-between goes to infinity, the devices are isolated from one another in terms of interference and the interference intensity functions become zero, which means Equation (2) reduces to the SI demand curve.

With the interference intensity functions, firm *i*'s demand curve becomes

$$P_i = \frac{a}{1+(K_i q_i + \sum_j K_j \frac{q_j}{d_{ij}})/c} - b(q_i + Q_{-i}) \tag{3}$$

In this paper, we assume that the number of terms in the summation depends on *c*. In other words, in addition to attenuating the magnitude of interfering signals, devices that are more robust also reduce spectral distance for which interference is an issue so if the spectral distance between firms is fixed, more robust devices reduce the number of perceived interfering systems. On the other hand, given the total bandwidth allocated to the service (let this be *L*) when the number of firms in the market, *n*, gets larger, the separation between systems is smaller and devices in the adjacent band will have a stronger interference effect on firm *i*'s devices.[7] It follows that for firm *i*, the number of perceived interfering systems (the number of terms in the summation) is a function of *n* and *c* and will be determined endogenously.

Recognizing this, we replace d_{ij} with $\frac{mL}{n}$, which is the spectral distance between the operating frequencies of firm *i* and firm *j* who is in the *m*th adjacent sub-band to *i*. And the demand curve becomes

$$P_i = \frac{a}{1+(K_i q_i + 2\sum_{m=1}^{\hat{m}} K_{i+m} q_{i+m} \frac{n}{mL})/c_i} - bq_i - 2b\sum_{m=1}^{\hat{m}} q_{i+m} - b\sum_{|l|>\hat{m}+1} q_l \tag{4}$$

where $\hat{m}$ is the number of perceived interfering systems who are within the spectral distance where interference is an issue ($\hat{m}$ depends on the spectral distance between firms and *c*), and the last term is the price effect of devices who are too far away spectrally to interfere with device *i*.

V. Profit Function Utilized for Welfare Comparison

With the demand function from the previous section, we can set up the profit-maximizing conditions for firms. Assume the fixed cost each firm incurs is *F*. The demand function (4) then yields the following profit function, which all firms in the market seek to maximize:

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[7] Actually, the firms are not facing identical situations because the firms operating at the boundaries of the entire PR spectrum face different interference effects from within/without the bands. We address this end point problem by assuming firms can costlessly adjust their operating frequency and making the circular city assumption used in Salop's paper on product differentiation (Salop, S., 1979, "Monopolistic competition with outside goods" Bell Journal of Economics, 10, 141-156).

$$\begin{aligned}\pi_i &= (P_i - c_i)q_i - F \\ &= \frac{aq_i}{1 + (K_i q_i + 2\sum_{m=1}^{\hat{m}} K_{i+m} q_{i+m} \frac{n}{mL}) / c_i} - bq_i^2 - 2b\sum_{m=1}^{\hat{m}} q_{i+m} q_i - b \sum_{|l| > \hat{m}+1} q_l \, q_i - c_i q_i - F \end{aligned} \tag{5}$$

Depending on the setup of the entry condition, this profit function can describe a wide range of spectrum governance regimes from open commons to exclusive property rights regime. When there is no entry restriction, *n* will keep increasing until firms' profits drop to zero and entry stops, this is the open commons scenario. We can also have an exclusive property rights regime when the number of firms is sufficiently small that their signals are entirely separated in the frequency domain ($\hat{m} = 0$); we call this the *zero-interference assignment* (ZIA).[8] And we can also have intermediate cases where the number of firms allowed to enter is larger than that under ZIA so that firms have weaker protection from interference.

On the other hand, when the government does not set a mandated level of interference robustness, the c_i's in Equation (5) will be a firm-specific variable which the firms can adjust to maximize their profits. But with a government mandated level of interference robustness, the c_i's reduce to an exogenous variable that all firms take as a given.

These two important policy instruments (entry condition and mandated level of robustness) allow us to organize all these possibilities into the following four regimes:

(A) Limited entry with mandated level of interference robustness: Government determines *n* and c_i; firms set q_i's to maximize their profits

(B) Open entry with mandated level of interference robustness: Government determines *c*; firms set q_i's to maximize their profits; number of firms in the market determined by the zero-profit condition

(C) Limited entry without mandated level of interference robustness: Government determines *n*, firms set q_i's and c_i's to maximize their profits

(D) Open entry without mandated level of interference robustness: firms set q_i's and c_i's to maximize their profits; number of firms in the market determined by the zero-profit condition

Given firms' control variables, we can obtain market equilibrium and welfare outcome by solving the first order conditions for profit maximization. Obviously, this is not a straight-forward task since the number of terms in the summation depends on *c* (see Appendix I). A numerical method is developed to obtain the solution.

VI. Results

This section presents and compares the results of the four scenarios described in Section V.

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[8] The ideas of "signals entirely separated in the frequency domain" and "zero-interference" should not be taken literally. Rather, these are relative concepts depending on the noise floor.

In order to study the properties of these regimes numerically we had to choose certain parameter values. The parameters used to generate these outcomes are $a = 100$, $b = 1$, $F = 10$, $K_1 = K_2 = 1$. Also the total bandwidth allocated to this service is assumed to be 10 MHz and each system occupies 2 MHz of bandwidth, indicating that the ZIA allows up to five firms in the market; any assignment allowing greater than five firms can only provide weaker interference protection.

Scenario A: Controlled Entry with Mandated Level of Interference Robustness

Figure 2 shows welfare result of a regime with controlled number of firms and mandated level of interference robustness. The result is a family of welfare curves indexed by n, with each curve showing how welfare varies with the mandated level of interference robustness at a given number of firms. We can find the welfare optimum in this case and use it as a benchmark to see how well other regimes can do.

As discussed at the end of Section IV, the demand curve each firm faces also depends on the number of interfering systems the firm perceives, which in turn depends on how robust devices are. This leads to the discontinuities in the welfare curves: when the mandated interference robustness is raised beyond a certain level, the number of perceived interferers suddenly drops,[9] and welfare jumps up due to this sudden drop in interference. This is reflected by shifting to a higher welfare curve associated with the reduced number of perceived interferers, and as mandated interference robustness keeps increasing there will be more such jumps until the number of perceived interferers drops to zero.[10]

With any given number of firms, the shape of the welfare curve is skewed to the left, suggesting that as the devices are relatively dumb, increasing interference robustness can significantly improve welfare but the effect soon hits a plateau and starts to decline due to reduced sales and surplus associated with increasing costs.

The upward shift of the curves in Figure 2 is not monotonic. Starting from zero, increases in the number of firms increase welfare while skewing the curve further to the left. However, this upward trend peaks at $n = 57$, where welfare maximum is 2,512[11] (when the government sets the mandated level of interference robustness at around 10). As the number of firms allowed to enter the market exceeds 57, the welfare increase brought about by the additional firms no longer outweighs the additional fixed costs and the welfare curves start to shift downwards.

Given that the maximum number of firms under the ZIA is 5, this graph suggests that the government can improve social welfare by allowing more entry (global optimum welfare level 2,512 at $n = 57$) than that allowed under the ZIA (maximum welfare level 1,220 at $n = 5$).

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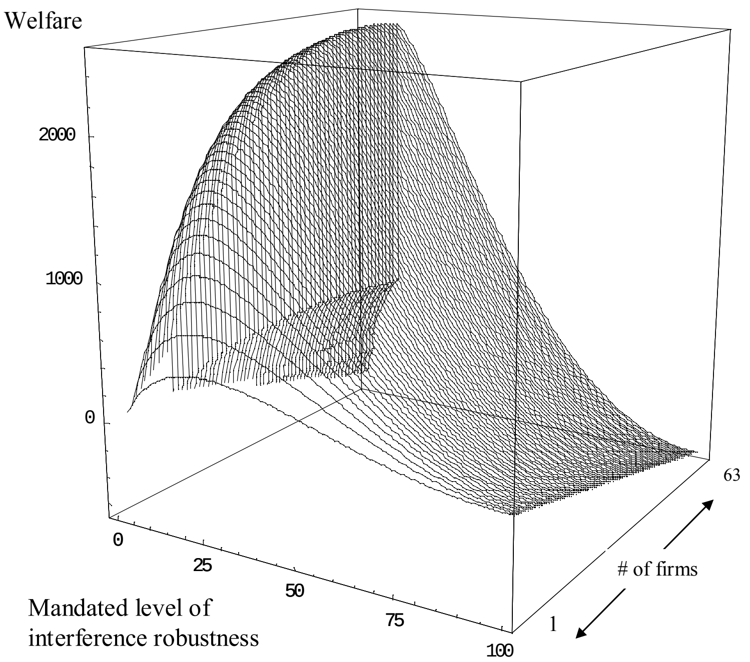
[9] These are discrete drops, since the number of interferers has to be an integer.

[10] However, these welfare jumps are quite small relative to the overall level of welfare that the welfare functions seem continuous for the most part in Figure 2, except in cases where the number of firms is large and the interference robustness and cost are low (left in Figure 2), when the effect of increased interference robustness is most pronounced.

[11] This is an index number.

Figure 2 Welfare outcomes in a property rights regime with mandated minimum level of interference robustness



It is note-worthy that although the global optimum allows more firms to enter the market than that allowed by ZIA, it does not mean at the optimum users are suffering from high level of interference, because the equilibrium properties show that at the optimum firms produce devices with robustness such that the number of perceived interferers is zero. In other words, at the global optimum, devices' interference robustness has to be set high enough that users perceive no interference from devices operating in adjacent bands.

Scenario B: Open Entry with Mandated Level of Interference Robustness

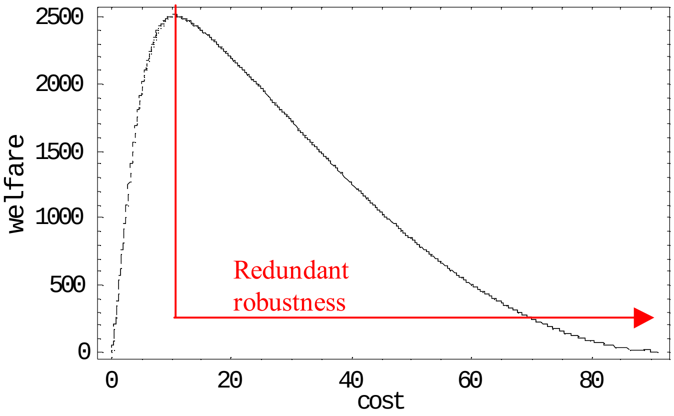
When there is no entry restriction, the equilibrium outcomes are subject to the zero-profit assumption. Whenever an additional firm can profitably enter the market, the number of firms will keep increasing and this will lead to changing demand functions and consequently change the welfare function.

Figure 3 shows the results for the scenario of open entry (open commons) with mandated level of interference robustness. This actually is a piecewise continuous function consisting of segments of the welfare curves derived in the previous scenario.

The mandated level of interference robustness has an effect not only on the number of perceived interferers and demand, but also on the number of firms operating in the market. As the mandated interference robustness goes up, the concomitant higher cost reduces profitability and gradually firms experiencing negative profits will be forced to exit the market. This effect changes welfare through reducing sales and number of firms, reflected by a downward shift of the welfare curve. To understand this, let us first assume that the social planner requires such robust devices that profitability is reduced by the high cost to the point where only one firm can operate profitably in the market. If the mandated level of interference robustness goes down, cost will go down and profits will increase; at some point a second firm will find it profitable to enter the market and the relevant robustness-welfare schedule is the one with 2 firms. This process is repeated as the level of interference robustness keeps going down and more firms enter the market. The curve in Figure 3 is a piece-wise continuous one since the relevant demand and welfare curves change with the level of interference robustness, but the jumps are so small compare to the overall values of welfare at those discontinuities that they seem continuous.

In this study, welfare peaks at 2,488, when the mandated level of interference robustness is set at 12 and the accompanying number of firms is 63. At this point of interference robustness, the devices also have the property that they are smart enough to prevent devices operating in adjacent bands from perceivably affecting each other. This result has some interesting implications: If the social planner's goal is to maximize static welfare through setting some minimum level of interference robustness, the mandated interference robustness level should not be set too high otherwise there will be excess robustness and welfare goes down due to the wasteful robustness requirement.

Figure 3 Welfare outcomes in an open commons regime with mandated minimum level of interference robustness



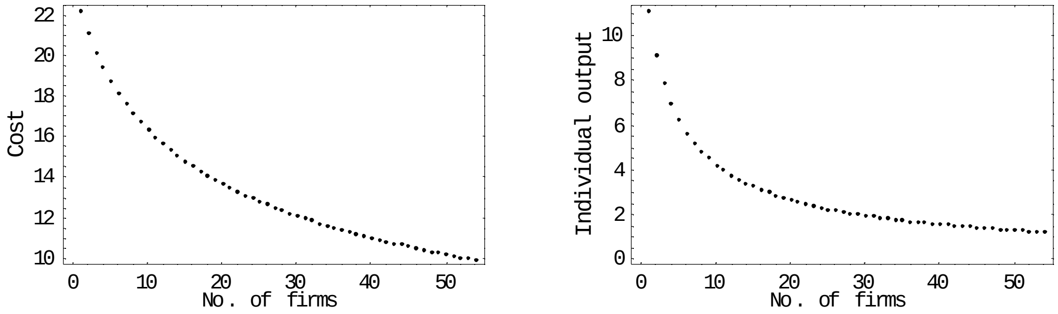
Compared with optimum in scenario A, this result shows that in terms of optimal static welfare, controlling both the number of firms and mandated level of interference robustness and only control interference robustness do not yield much different welfare outcomes. Also, the welfare optimum under the open commons regime with mandated level of interference robustness is greater than that achieved under the ZIA regime (1,220 at $n = 5$).

Scenario C: Controlled Entry without Mandated Level of Interference Robustness

Under this scenario, firms can set both output level and the level of interference robustness. A similar finding in this scenario that all these equilibria are situations where firms produce devices with such robustness that devices operating in adjacent bands do not perceivably affect with each other.

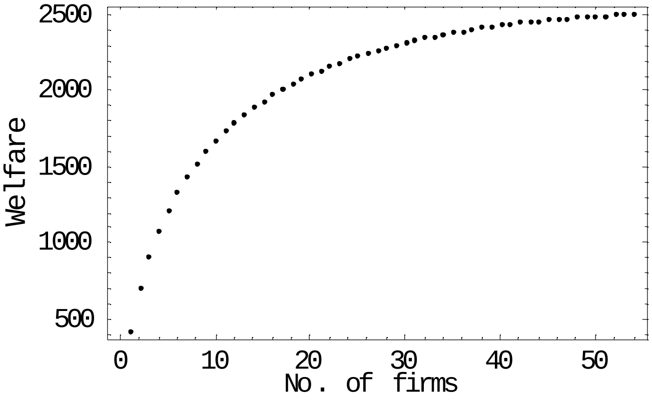
As the number of firms increases, the number of units sold per firm falls, which is not surprising, and interference robustness falls as well, which is more surprising (Figure 4). Since in all these equilibria devices are robust enough to keep different firms' devices from interfering with each other, the observation that firms' chosen level of interference robustness goes down as the number of firms increases is likely to have more to do with the diminishing incentive to reduce interference among a firm's own devices as the number of firms sharing the market increases.

Figure 4 Individual output and interference robustness levels as functions of number of firms in the market



With the selected values of parameters, the ZIA corresponds to situations where $n \leq 5$. Obviously, without mandated level of interference robustness, the ZIA property rights regime also does not do as well as that with weak protection in terms of static welfare. (At n = 54, welfare is 2,507 as opposed to 1,220 at n=5.)

Figure 5 Welfare outcome in situations where firms can set both output level and devices' interference robustness



As discussed earlier, when n gets so large that firms earn zero-profits, the equilibrium with this scenario becomes same as that for an open commons with no mandated level of interference robustness regime (scenario D); however, in this case and many others, when the number of firms allowed to enter the market exceeds 54, there will be no stable simultaneous-move symmetric equilibria. Whether there will be equilibria depends on the fixed cost. If fixed cost is sufficiently high, the number of firms the market can support will be smaller and this issue will no longer be relevant. For example, the fixed cost can be high enough that 50 is the maximum number of firms the market can support; in that case we will see the welfare increase and converge to that under the open commons regime as the number of firms approaches the number producing zero profits ($n = 50$ in that case).

Scenario D: Open Entry without Mandated Level of Interference Robustness

This is the logical equivalent to the equilibrium for scenario C when the number of firms is increased to the point where firms earn zero profits. So similar to scenario C where the number of firms is greater than 54, the parameter values we pick produce no equilibrium under this regime—although this may not always be the case.

The welfare trend in Figure 5 suggests that, when the open commons equilibrium falls on the top-right corner, it yields greater welfare than all the other equilibria under scenario C, and we can be sure that it will yield higher static welfare than that under scenario C. However, this just reflects our work to this point, which uses a model with particular level of fixed cost. Preliminary analysis suggests that, stable simultaneous-move symmetric equlibria can emerge again beyond the region where such equilibria do not exist. In that case, as the number of firm keeps increasing, the number of perceived interferers each firm faces increases as well; as a result, beyond the region with no simultaneous-move symmetric equilibria, welfare will be lower and the result is a sub-optimal outcome.

VII. Conclusions

In summary of all four scenarios, this paper's major findings are as follows. First, with or without mandated level of interference robustness, a property rights regime with strong interference protection band assignment (by separating firms' operating frequencies sufficiently far apart) cannot compete in static welfare with either the open commons regime or a regime where there are more spectrum owners with weaker protection against interference from neighboring firms.

Second, all the welfare optima require devices that are robust enough to distinguish devices operating in adjacent bands, but whether this property holds for all equilibria, including those yield sub-optimal welfare outcomes, depends on the fixed cost.

Third, not surprisingly, when a policymaker has control over both the number of firms entering the market and devices' interference robustness, it can achieve the highest welfare outcome (scenario A). And the welfare optima under all four scenarios are quite close.

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Appendix I: Solving For Equilibria Under Different Governance Regimes

Equation (5) gives individual firms' profit function:

$$\pi_i = (P_i - c_i)q_i - F$$

$$= \frac{aq_i}{1 + (K_i q_i + 2\sum_{m=1}^{\hat{m}} K_{i+m} q_{i+m} \frac{n}{mL})/c_i} - bq_i^2 - 2b\sum_{m=1}^{\hat{m}} q_{i+m} q_i - b\sum_{|l|>\hat{m}+1} q_l\, q_i - c_i q_i - F \qquad (5)$$

The first order profit-maximizing condition with respect to q_i and c_i are:

$$\frac{\partial \pi_i}{\partial q_i} = \frac{acL(cL + 2\sum_{m=1}^{\hat{m}} K_{i+m} q_{i+m} \frac{n}{m})}{(Lc + LK_i q_i + 2\sum_{m=1}^{\hat{m}} K_{i+m} q_{i+m} \frac{n}{m})^2} - c + b(-2q_i - 2\hat{m}q_{i+m} + q_l - nq_l + 2\hat{m}q_l) = 0 \qquad \text{and}$$

$$\frac{\partial \pi_i}{\partial c_i} = \frac{aq_i L(K_1 L q_i + 2\sum_{m=1}^{\hat{m}} K_{i+m} q_{i+m} \frac{n}{m})}{(Lc + LK_i q_i + 2\sum_{m=1}^{\hat{m}} K_{i+m} q_{i+m} \frac{n}{m})^2} - q_i = 0$$

Since the firms are identical, in equilibrium $q_i = q_{i+m} = q_l$ for all i, j, and l. Also, since the rivals are identical, all the K_{i+m}'s should be the same, denoted as K_2. Given symmetry, we can let firm 1 be the representative firm.

Now the 1st order conditions become

$$\frac{\partial \pi_i}{\partial q_i} = \frac{ac_i L(c_i L + 2\sum_{m=1}^{\hat{m}} K_{i+m} q_{i+m} \frac{n}{m})}{(Lc_i + LK_i q_i + 2\sum_{m=1}^{\hat{m}} K_{i+m} q_{i+m} \frac{n}{m})^2} - c_i - b(1+n)q_i = 0 \;\ldots\ldots\ldots(6)$$

and

$$\frac{\partial \pi_i}{\partial c_i} = \frac{aq_i L(K_1 L q_i + 2\sum_{m=1}^{\hat{m}} K_{i+m} q_{i+m} \frac{n}{m})}{(Lc_i + LK_i q_i + 2\sum_{m=1}^{\hat{m}} K_{i+m} q_{i+m} \frac{n}{m})^2} - q_i = 0 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(7)$$

Along with the zero profit constraint ($\pi_i = 0$), these first order profit maximizing conditions can jointly determine the equilibria under the four regimes discussed in Section V. For example, for scenario (A), we solve (6) for $q_i(n,c)$, and for each n, we can derive $q_i(c)$. Plugging this back to (4) and (5) we will have prices and profits as functions of c for various n's. These functions can then be used to derive social welfare (defined as the sum of consumer surplus and producer surplus net of fixed costs) as a function of c for any given n. For scenario (B), we only need to add the zero profit constraint.

For scenarios (C) and (D), we need to solve (6) and (7) simultaneously because firms control both variables. This will give the equilibrium q_i and c_i as functions of n. With q_i, c, and n, we will be able to find out the social welfare and individual profits associated with each n via the same method just described above. The results constitute a spectrum of outcomes ranging from the open commons regime (the case where an additional firm will lead to negative profits for firms) and the property rights regime (where the number of firms are set at a lower level to ensure lower mutual interference).

Obviously, solving (6), (7), and the zero profit constraint for the equilibrium quantity is not so straight-forward since the number of terms in the summation depends on c. We use a numerical method to derive the results for various values of n and Mathematica is used to solve the nonlinear equations.